ch : Electric Potential & Capacitance

Electric potential :- Electric potential at a point can be defined as the work done in bringing a unit positive charge from ∞ to that point.

→ If W J of work is done in bringing q C of charge from ∞ to a point, then the potential at that point,

$$\boxed{V = \frac{W}{q}}$$

→ Potential is a scalar quantity & its unit is volt.

$$1 \text{ volt} = \frac{1 \text{ Joule}}{1 \text{ Coulomb}}$$

∴ Electric potential at a point is said to be 1 Volt if 1 Joule of work is done in bringing 1 C of charge from ∞ to that point.

→ **Electric Potential Difference** :- Electric potential diff. b/w any 2 points can be defined as the work done in bringing a unit +ve charge from one point to the other.

→ If W_{AB} is the work done in bringing a charge q from A to B Then the potential diff. b/w B & A,

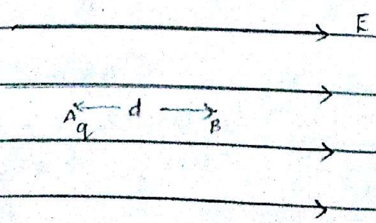
$$V_B - V_A = \frac{W_{AB}}{q}$$

Illy, $V_A - V_B = \frac{W_{BA}}{q}$

→ The unit of potential difference is Volt.

Relation b/w Electric field & Potential :-

Consider 2 points A & B separated by a distance d in a uniform Electric field of intensity E.



The force experienced by a charge q kept at the point A.

$F = qE$

∴ The work done in moving the charge from A to B.

$$W_{AB} = Fd$$

$$W_{AB} = qEd$$

∴ The potential diff. b/w B & A

$$V_B - V_A = \frac{W_{AB}}{q}$$

$$= \frac{qEd}{q}$$

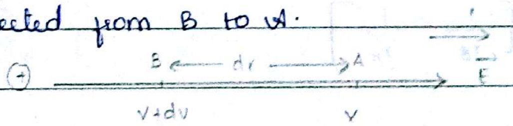
$$V_B - V_A = Ed$$

∴ The potential diff., $V_d = Ed$

$$\boxed{E = \frac{V_d}{d}}$$

→ Electric field is the negative potential gradient :-

Consider a points A & B separated by a small distance dr, in a non uniform ele-ctric field directed from B to A.



Let v & v+dv be the electric potentials at A & B resp.

Then, dv = Potential diff b/w A & B

= work done per unit charge from A to B (W = F·S)

= Force per unit charge · displacement from A to B (E = F/q)

$$dv = \vec{E}\cdot\vec{dr}$$

$$= Edr\cos\theta$$

$$= Edr\cos 180$$

$$dv = -Edr$$

$$\boxed{E = \frac{-dv}{dr}}$$ ie Electric field is the -ve potential gradient

→ Electric potential due to a point charge :-

Consider a point P at a distance R from a point charge q. The intensity of electric field at a point P is given by,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

A & B are 2 points at distances r_A & r_B resp. from q

Let dv be the potential diff b/w A & B.

$$E = \frac{-dv}{dr}$$

$$dv = -E\,dr$$

$$\int_A^B dv = -\int_A^B E\,dr$$

$$\int \frac{1}{r^2}\,dr$$

$$[V]_A^B = -\int_A^B \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr$$

$$\int \frac{r^{-2+1}}{-2+1}\,dr$$

$$V_B - V_A = \frac{-q}{4\pi\varepsilon_0}\int_A^B \frac{1}{r^2}\,dr$$

$$= \frac{r^{-1}}{-1}$$

$$= -\frac{1}{r}$$

$$V_B - V_A = \frac{-q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_A^B$$

$$V_B - V_A = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]$$

Let the point A be at ∞

$\therefore r_A = \infty$ & $V_A = 0$.

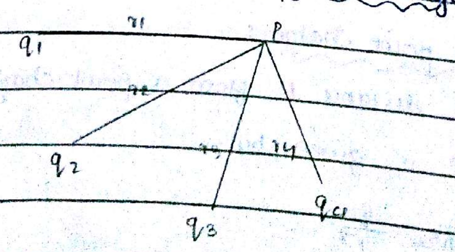
$$\therefore V_B - 0 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{\infty}\right]$$

$$V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_B}\right]$$

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_B}$$

$\therefore$ The Electric potential at P, $\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}}$

Potential due to a no: of point charges



$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$$

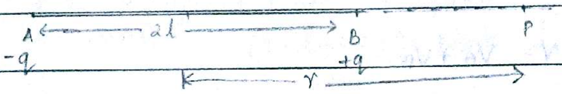
$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$V_4 = \frac{1}{4\pi\varepsilon_0} \frac{q_4}{r_4}$$

$$\therefore \quad V = \underline{\underline{V_1 + V_2 + V_3 + V_4}}.$$

Electric potential due to a dipole

i) At a point on the axial line



$$V_A = \frac{1}{4\pi\varepsilon_0} \frac{-q}{(r+l)} \qquad V_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-l)}$$

$$V = V_A + V_B$$

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-q}{(r+l)} + \frac{1}{4\pi\varepsilon_0} \frac{q}{(r-l)}$$

$$V = \frac{q}{4\pi\varepsilon_0} \left(\frac{-1}{(r+l)} + \frac{1}{(r-l)} \right)$$

$$V = \frac{q}{4\pi\varepsilon_0} \left(\frac{-(r-l) + r+l}{r^2 - l^2} \right)$$

$$V = \frac{q}{4\pi\varepsilon_0} \left(\frac{-r+l+r+l}{r^2 - l^2} \right)$$

$$V = \frac{q}{4\pi\varepsilon_0} \left(\frac{2l}{r^2 - l^2} \right)$$

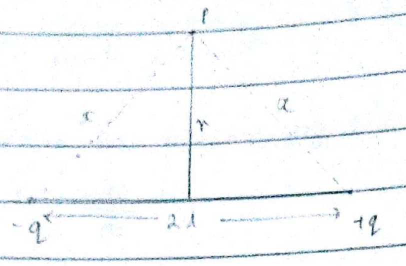
$$V = \frac{q}{4\pi\varepsilon_0} \times \frac{2l}{r^2 - l^2} \qquad (q \times 2l = P)$$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{P}{(r^2 - l^2)}$$

If $r >> l$, l^2 can be neglected.

$$\therefore \boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^2}}.$$

ii) At a point on equatorial line :-



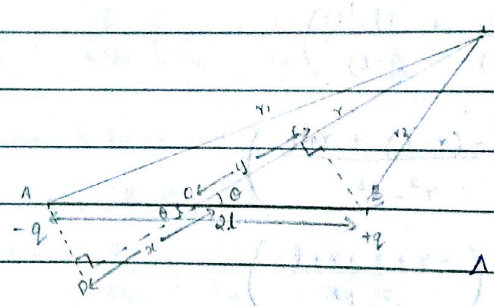
$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{-q}{x} \qquad V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{x}$$

$$V = V_A + V_B$$

$$= 0$$

Electric potential due to a dipole :-

Consider an electric dipole consisting of 2 charges $-q$ & $+q$ separated by a distance $2l$. P is a point at a distance r from the centre of the dipole. r makes an angle θ with the dipole moment



In $\triangle AOD$

$$\cos\theta = \frac{x}{l}$$

$$x = l\cos\theta$$

$\triangle OCB$,

$$\cos\theta = \frac{y}{l}$$

$$y = l\cos\theta$$

Potential at P due to a charge $-q$.

$$V_A = \frac{1}{4\pi\varepsilon_0} \times \frac{-q}{r_1}$$

Potential at P due to a charge $+q$.

$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$$

$\therefore$ Net potential at P, $V = V_A + V_B$.

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{-q}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(-\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{-r_2 + r_1}{r_1 r_2}\right)$$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r_1 - r_2}{r_1 r_2}\right) \longrightarrow (1)$$

From fig, $r_1 \approx r + x$

$r_1 = r + l\cos\theta$

IIIly $r_2 \approx r - y$

$r_2 = r - l\cos\theta$

$\therefore$ (1) $\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{r + l\cos\theta - r + l\cos\theta}{(r + l\cos\theta)(r - l\cos\theta)}\right)$

$$V = \frac{q}{4\pi\varepsilon_0}\left(\frac{2l\cos\theta}{r^2 - l^2\cos^2\theta}\right)$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos\theta}{r^2}} \quad (\because r >> l)$$

where $p = q \times 2l$, the dipole moment.

Special cases :-

Case I :- Let $\theta = 0°$ ie the point P is on the axial line.

Then, $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos 0}{r^2}$

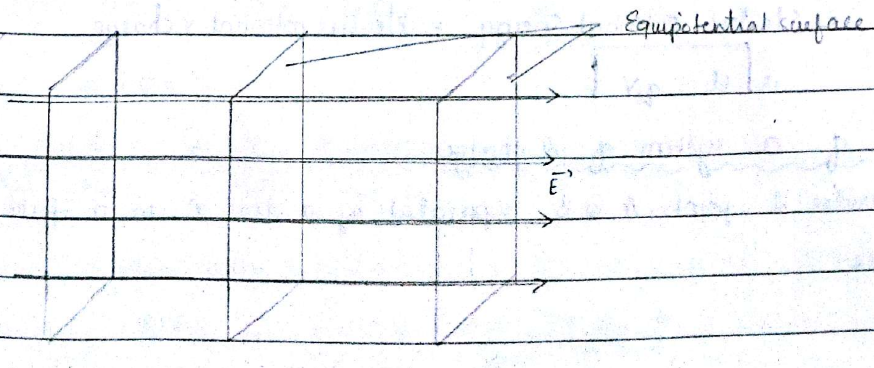
$$V = \frac{1}{4\pi\varepsilon_0}\frac{P}{r^2}$$

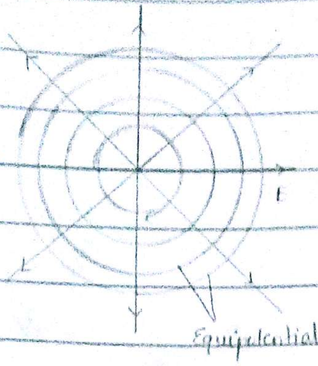
Case II :- Let $\theta = 90°$, ie the point P is on the equatorial line.

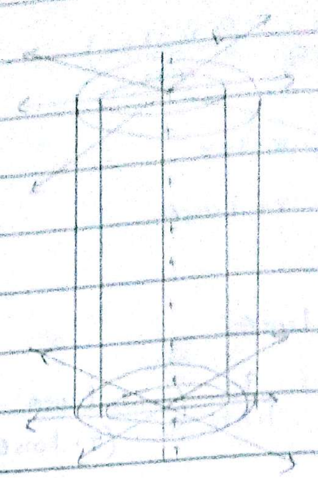
Then $V = \frac{1}{4\pi\varepsilon_0}\frac{P\cos 90}{r^2}$

$$V = 0$$

Equipotential Surface :- A surface on which the electric potential is the same everywhere is called an equipotential surface.







Properties :- → The potential diff· b/w any 2 points on an equipotential surface is 0.

→ The work done in moving a charge from one point to another on an equipotential surface is 0.

→ Electric field is always ⊥r to the equipotential surface.

Proof :- We have, $\vec{E} \cdot \vec{dr} = dv$

On an equipotential surface, $dv = 0$

$\therefore \vec{E} \cdot \vec{dr} = 0$

$E\,dr\cos\theta = 0$

$\cos\theta = 0$

$\therefore \theta = 90°$

→ 2 equipotential surface will never intersect :- If they intersect at a point, there will be 2 diff· potentials on a surface which is impossible.

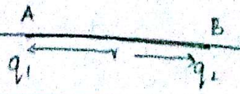
Electric Potential Energy :- Electric potential Energy of a charge at a point can be defined as the work done in bringing that charge from ∞ to that point.

$\therefore$ Electric potential Energy = Electric potential × charge.

ie $\boxed{U = qV}$

PE of a system of 2 charges :-

Consider 2 points A & B separated by a dist· r in a field free space.



Work done in bringing a charge q_1 to the point A

$W_1 = 0$

due to this charge, there will be a potential at the point B & is given by

$$V = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r}$$

∴ The work done in bringing a charge q_2 to the point B,

$$W_2 = V q_2$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

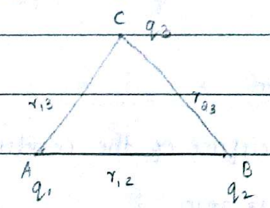
The total work done in forming the syst, $W_{tot} = W_1 + W_2$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

This work done is stored as the potential Energy of the system.

$$\boxed{U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}}$$

Potential Energy of a system of 3 charges :-



$W_1 = 0$

$$V_B = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}}$$

$$W_2 = V_B q_2$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}$$

$$V_c = \frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0} \frac{q_2}{r_{23}}$$

$$W_3 = V_c \times q_3$$

$$V_c = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0} \frac{q_2 q_3}{r_{23}}$$

$$W = W_1 + W_2 + W_3$$

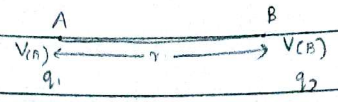
$$\frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_3}{r_{13}} + \frac{1}{4\pi\varepsilon_0} \frac{q_2 q_3}{r_{23}}$$

$$W = \frac{1}{4\pi\varepsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

$$\therefore \quad U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

Potential Energy in an External field :-

Let $V_{(A)}$ & $V_{(B)}$ be the electric potentials at points A & B resp.



$$W_1 = V_{(A)} q_1$$

$$W_2 = V_{(B)} q_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$\therefore \quad \boxed{U = V_{(A)} q_1 + V_{(B)} q_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}}$$

Electron Volt (eV) :- Electron Volt is a unit of energy. 1eV can be defined as the energy acquired by an $\bar{e}$ when its is moving under a potential difference of 1 Volt.

$$1\,eV = 1.6 \times 10^{-19}\,J.$$

Electrostatic properties of a conductor :-

i) Charge will remain only on the surface of the conductor

ii) Electric field inside a conductor is zero

iii) Electric potential inside a conductor is a constant and is equal to the potential on the surface.

We have,

$$E = \frac{-dv}{dr}$$

Inside a conductor, $E = 0$

$$\therefore \quad \frac{dv}{dr} = 0.$$

$$v = \text{a constant}.$$

iv) Electric field is always ⊥r to the surface of a conductor.

→ If it is not ⊥r, there will be a tangential component for the electric field. That will produce current in a conductor without any source which is impossible

Electrostatic Shielding :- It is a process of creating a region which is free from all the electrical effects. It works on the principle that the electric field inside a conductor is zero. It can be used to protect electronic devices from the effects of lightening.

Q. 3 identical charges q C each are kept at point A, B & C as shown in the fig. Find the work done in moving the charge q from B to the point D along the semicircle of radius r.

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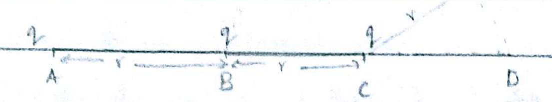


Acc. to work-energy theorem, work done = change in Energy

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_2 \cdot q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right]$$

$$U_1 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{r} + \frac{q^2}{r} + \frac{q^2}{2r}\right]$$

$$U_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{3r} + \frac{q^2}{2r} + \frac{q^2}{r}\right]$$

$$\cancel{U_1 = \frac{q^2}{4\pi\varepsilon_0}\left[\frac{2+2+1}{2}\right.}$$

$$W = U_1 - U_2$$

$$W = \frac{1}{4\pi\varepsilon_0} q^2 \left[\frac{1}{r} + \frac{1}{r} + \frac{1}{2r} - \frac{1}{3r} - \frac{1}{2r} - \frac{1}{r}\right]$$

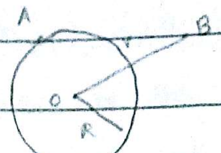
$$W = \frac{1}{4\pi\varepsilon_0} q^2 \left[\frac{1}{r} - \frac{1}{3r}\right]$$

$$\frac{1}{4\pi\varepsilon_0} q^2 \left[1 - \frac{1}{3}\right]$$

$$\frac{1}{4\pi\varepsilon_0} q^2 \left[\frac{2}{3}\right]$$

$$W = \frac{2q^2}{12\pi\varepsilon_0}$$

Q. A metallic sphere with O as the centre & R as the radius is having a surface charge density, σ. Find the electric field & electric potential at the points O, A & B



From Gauss theorem

application,

$E_O = 0.$ $\left(\frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}\right)$ $q = \sigma 4\pi R^2$

$E_A = \frac{\sigma}{\varepsilon_0}$

$E_B = \frac{\sigma}{2\varepsilon_0} \times \frac{R^2}{r^2}$

$V_O = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

$\frac{1}{4\pi\varepsilon_0} \frac{\sigma 4\pi R^2}{R}$

$= \frac{\sigma R}{\varepsilon_0}$

$V_A = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

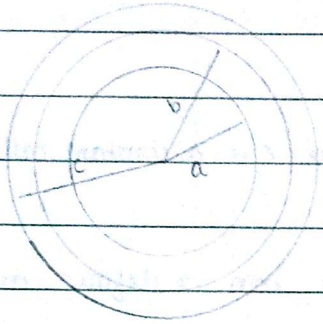
$= \frac{\sigma R}{\varepsilon_0}$

$V_B = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

$= \frac{1}{4\pi\varepsilon_0} \times \frac{\sigma 4\pi R^2}{r}$

$= \frac{\sigma R^2}{\varepsilon_0 r}$

Q. A, B and C are 3 concentric spheres with Surface charge densities $+\sigma$, $-\sigma$, $+\sigma$ resp. The potential on the sphere A is equal to the potential on the sphere C. Find the relation b/w a, b & c



$q_A = \sigma \times 4\pi a^2$

$q_B = -\sigma \times 4\pi b^2$

$q_C = +\sigma \times 4\pi c^2$

Potential at A = Potential at C.

$$V_A = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{a} + \frac{q_B}{b} + \frac{q_C}{c}\right]$$

(other charges can exert a potential on A).

Illy
$$V_C = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_A}{c} + \frac{q_B}{c} + \frac{q_C}{c}\right]$$

$V_A = V_C$

$$\therefore \frac{1}{\cancel{4\pi\varepsilon_0}}\left[\frac{q_A}{a} + \frac{q_B}{b} + \cancel{\frac{q_C}{c}}\right] = \frac{1}{\cancel{4\pi\varepsilon_0}}\left[\frac{q_A}{c} + \frac{q_B}{c} + \cancel{\frac{q_C}{c}}\right].$$

$$\frac{q_A}{a} + \frac{q_B}{b} = \frac{q_A}{c} + \frac{q_B}{c}$$

$$\frac{\sigma \times 4\pi a^2}{a} + \frac{-\sigma \times 4\pi b^2}{b} = \frac{\sigma \times 4\pi a^2}{c} + \frac{-\sigma 4\pi b^2}{c}.$$

$$\sigma 4\pi a - \sigma 4\pi b = \sigma 4\pi \left(\frac{a^2 - b^2}{c}\right)$$

$$\sigma \cancel{4\pi}\,\cancel{(a-b)} = \frac{\sigma \cancel{4\pi}\,(a+b)\cancel{(a-b)}}{c}\; c$$

$$\underline{\underline{c = a+b}}$$

CAPACITOR :- Capacitor is a device used to store electric charges

CAPACITANCE (CAPACITY) :- When charge is given to a conductor its potential increases

ie $V \propto Q$

$$V = \frac{1}{C} Q$$

$$\boxed{C = \frac{Q}{V}}$$ where C is a constant called capacitance

→ Capacitance of a capacitor can be defined as the ratio of its charge to the potential

→ The unit of capacitance is Farad

→ **1 Farad** :- $1 \text{ Farad} = \frac{1C}{1 \text{ Volt}}$

→ The capacitance of a capacitor is said to be 1 Farad ~~when~~ if its potential is increased by 1 Volt when 1C of charge is given to it.

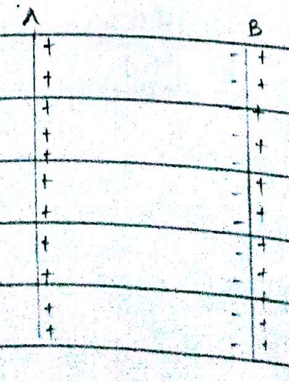
Note :- $1 \mu F = 10^{-6} F$

$1 pF = 10^{-12} F$

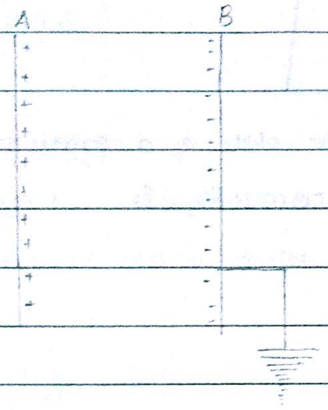
Principle of a Capacitor

Consider a plate A having a charge +Q. Let its potential be V. Then its capacitance, $C = \frac{Q}{V}$

This capacitance can be increased by decreasing the potential V. To reduce the potential of A, Bring another plate B, ∥ to A. -ve charges are induced on the inner side of the plate B & +ve charges are induced on the outer side as shown in the fig.



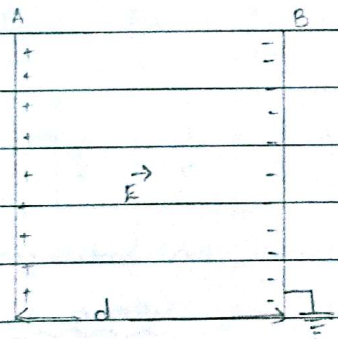
When the outer side of the plate B is earthed, the +ve charges will be discharg-ed to earth



∴ As a result of the -ve charge on the plate B, the net potential decreases & therefore the capacitance increases.

parallel plate capacitor :-

Consider a ‖^{l} plate capacitor consisting of 2 plates, each of area A & separat-ed by a distance d.



Let E be the electric field in b/w the plates. Then

$$E = \frac{\sigma}{\varepsilon_0}.$$

But $\sigma = \frac{Q}{A}$

$$\therefore E = \frac{Q}{A\varepsilon_0} \rightarrow ①$$

Let V be the potential diff. b/w the plates

Then, $E = \frac{V}{d} \rightarrow ②$.

From ① & ②

$$\frac{Q}{A\varepsilon_0} = \frac{V}{d}$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

$$\boxed{\text{capacitance, } C = \frac{\varepsilon_0 A}{d}}$$

Note: If the space b/w the plates of a capacitor is fully filled with a medium of relative permittivity, ε_r

Then $\boxed{C = \frac{\varepsilon_0 \varepsilon_r A}{d}}$ $\Rightarrow C = C_0 K$.

Q. The capacitance of a ‖ plate capacitance is C. Find its new capacitance if the area of the plates are doubled & the distance b/w them is halved.

$$C = \frac{\varepsilon_0 A}{d}$$

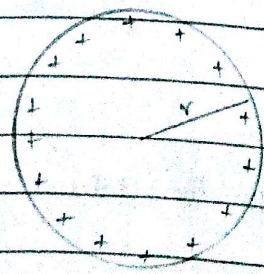
$$C' = \frac{\varepsilon_0 2A}{\frac{d}{2}}$$

$$C' = 4\frac{\varepsilon_0 A}{d}$$

$$\underline{\underline{C' = 4C}}$$

Capacitance of a spherical Capacitor:-

→ consider a spherical capacitor of radius r. Let Q be the charge given to it.



The potential on the sphere is given by,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

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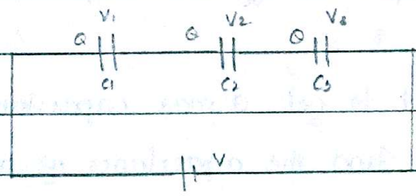
$$\frac{Q}{V} = 4\pi\varepsilon_0 r$$

$$\boxed{C = 4\pi\varepsilon_0 r}$$

Combination of capacitors

i) Capacitors in series :- Consider 3 capacitors of capacitance C_1, C_2 & C_3, connected in series across a battery / cell of EMF V Volt.



Let V_1, V_2 & V_3 be the Potential diff. across C_1, C_2, C_3 resp. Since the Capacitors are connected in series, The charge in them will be the same.

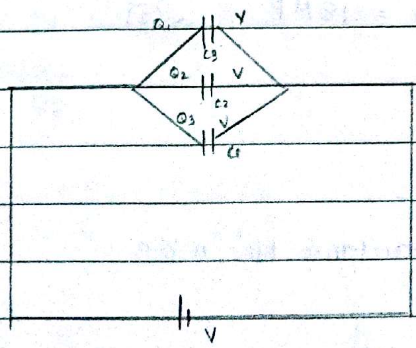
Then, $V = V_1 + V_2 + V_3$ $\qquad C = \frac{Q}{V}$

$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$ $\qquad \therefore V = \frac{Q}{C}$

$$\therefore \Rightarrow \boxed{\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

> ie when capacitors are connected in series, the effective capacitance decreases.

ii) Capacitors in parallel :- Consider 3 capacitors of capacitance C_1, C_2, & C_3, connected across a cell of EMF V Volt in parallel connection



Since the capacitors are connected in ||, the voltage across them are same.

Let Q_1, Q_2, Q_3 be the charges on C_1, C_2 & C_3 resp.

Then $\quad Q = Q_1 + Q_2 + Q_3$

But $\quad Q = CV$

$$CV = C_1V + C_2V + C_3V$$

or $$\boxed{C = C_1 + C_2 + C_3}$$

ie when capacitors are connected in ||el, the effective capacitance increases.

Q. You are given 3 capacitors C_1, C_2 & C_3 such that $C_1 \le C_2 < C_3$

i) How will you arrange these capacitors to get an effective capacitance less than the least among them - parallel

ii) Derive the expression for the effective capacitance.

Q. 2 capacitors are connected to get a max. capacitance of 9 MF & a min of 2 MF. Find the capacitance of both the capacitors.

$$C_1 + C_2 = 9\ \mu F \rightarrow ①$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2\ \mu F}$$

$$\frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2 \times 10^{-6}}$$

$$\frac{C_2 + C_1}{C_1 C_2} = \frac{1}{2 \times 10^{-6}}$$

$$9 \times 10^{-6} \times 2 \times 10^{-6} = C_1 C_2$$

$$18 \times 10^{-6} = C_1 C_2$$

or $$C_1 C_2 = 18\ \mu F \rightarrow ②$$

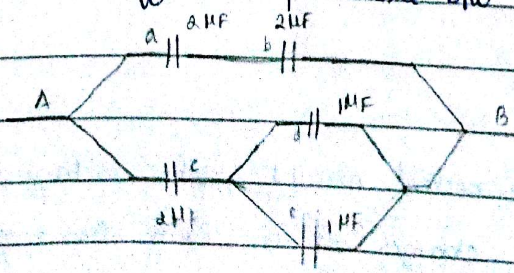
$$C_1 = 6\ \mu F$$

$$C_2 = 3\ \mu F$$

$6 \times 3 = 18$

$6 + 3 = 9$

Q. Calculate the effective capacitance b/w A & B.



$$\frac{1}{C_{ab}} = \frac{1}{2} + \frac{1}{2} = 1\ \mu F$$

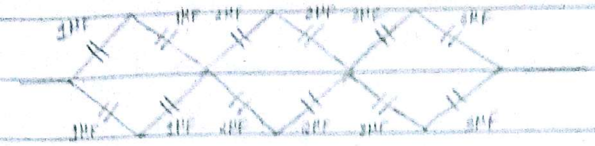
$C_{de} = 1+1 = 2$

$\frac{1}{C_{f}} = \frac{1}{2} + \frac{1}{2} = 1\ \mu F$

d & e - f

$\therefore$ The effective capacitance, $C = 1+1 = 2\ \mu F$

ii)



$\frac{1}{C} = \frac{1}{1} + \frac{1}{1} = 2$

$\frac{1}{C} = \frac{1}{2} + \frac{1}{2} = 1$

$\frac{1}{C} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$\frac{1}{C} = 2.$

$\therefore C_1 = \frac{1}{2} + \frac{1}{2} = 1$

$C_2 = 1+1 = 2.$

$\therefore C_3 = \frac{3}{2} + \frac{3}{2} = 3$

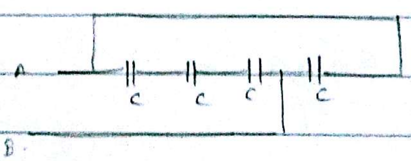
$$\frac{1}{C_{tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$

$$= \frac{6+3+2}{6} = \frac{11}{6}$$

$$\therefore C = \frac{6}{11}\ \mu F$$

iii)

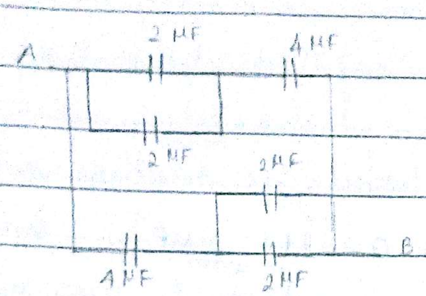


1st 3 capacitors - series.
1st 3 & 4th - parallel

$\frac{1}{C} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C}$

$\frac{1}{C} = \frac{3}{C}$ $\therefore C = \frac{C}{3}$

$C = \frac{C}{3} + C = \frac{4C}{3}$



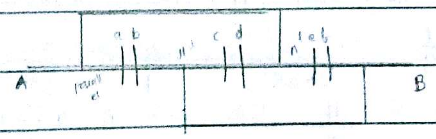
$C = 2+2 = 4$

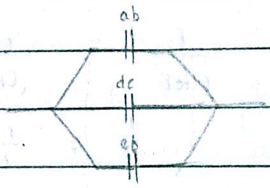
$\frac{1}{C} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\therefore C = 2$

$C = 4$

$\frac{1}{C} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ $\therefore C = 2$

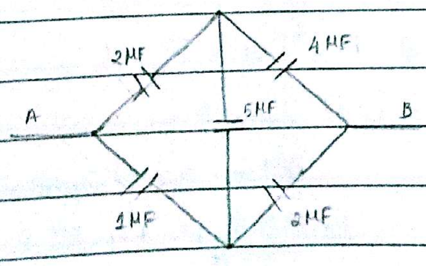
$\therefore C_{tot} = 2 + 2 = 4\,\mu F$

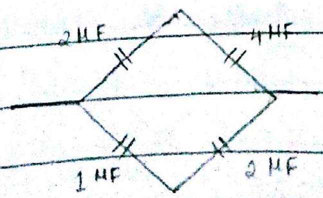




$C = c + c + c$

$= 3C$





(Acc. to Wheatstone's principle

$\frac{1}{C} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ $\therefore C = \frac{4}{3}$

$\frac{1}{C} = \frac{1}{1} + \frac{1}{2} = \frac{3}{2}$ $\therefore C = \frac{2}{3}$

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$C_{tot} = 2\ \mu F$

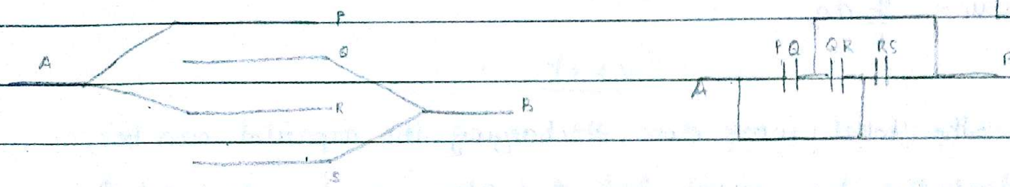
Q. 4 plates, each of area A are arranged as shown in fig. Find the capacitance b/w A & B

i)

$C = 2C$

$= \frac{2\varepsilon_0 A}{d}$

ii)



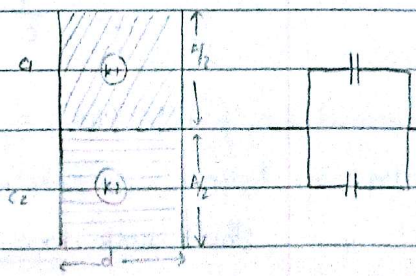
$C = 3C$

$\therefore C = \frac{3\varepsilon_0 A}{d}$

Q act as a common plate b/w P & R. So we can represent as ... plates ...

R act as a common plate b/w Q & S. So R can be represented 2 plates connected to each other.

Q. The capacitance of a capacitor of Plate area A & distance of separation d is c. Find the new capacitance if 2 medium of dielectric constants K_1 & K_2 are arranged or inserted in b/w the plates of capacitor as shown in the fig.



$C_1 = \frac{\varepsilon_0 K_1 A}{2d}$ $\quad C_2 = \frac{\varepsilon_0 K_2 A}{2d}$

$C = C_1 + C_2$

$= \frac{\varepsilon_0 K_1 A}{2d} + \frac{\varepsilon_0 K_2 A}{2d}$

$C_{tot} = \frac{CK_1}{2} + \frac{CK_2}{2}$

$C_{tot} = C\frac{(K_1+K_2)}{2}$

half of the plate is filled with 1 medium & other half of the plate is filled with other medium. So split the ...

Energy stored in a capacitor:

consider a capacitor of capacitance C. Let q be the charge on the capacitor at an instant of time. Then the potential diff b/w the plates of the capacitor, V =

$$V = \frac{q}{C}$$

Let V be the work done by per unit charge

$\therefore$ The small work done for a small charge dq is

$$dw = V dq$$

$$dw = \frac{q}{C} dq$$

$\therefore$ The total work done in charging the capacitor can be calculated by integrating the above equation from 0 to Q

ie $$w = \int_0^Q dw$$

$$w = \int_0^Q \frac{q}{C} dq$$

$$w = \frac{1}{C} \int_0^Q q\, dq$$

$$= \frac{1}{C} \left[\frac{q^2}{2} \right]_0^Q$$

$$= \frac{1}{C} \left[\frac{Q^2}{2} - 0 \right]$$

$$w = \frac{Q^2}{2C}$$

This work done is stored as the energy of the capacitor.

Therefore, $$\boxed{u = \frac{Q^2}{2C}}$$

But $Q = CV$

$$\therefore u = \frac{C^2V^2}{2C}$$

$$\boxed{u = \frac{1}{2} CV^2}$$

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But, $C = \frac{Q}{V}$

$$\therefore \boxed{U = \frac{1}{2} QV}$$

This energy is stored in the form of electric field in b/w the plates of the capacitor

→

Energy Density :- The Energy stored per unit Volume is called the Energy Density

ie $$\text{Energy Density} = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{\frac{1}{2} CV^2}{Ad}$$

$$= \frac{\frac{1}{2} \times \frac{\cancel{A} \varepsilon_0 V^2}{d}}{\cancel{A} d}$$

$$= \frac{1}{2} \varepsilon_0 \frac{V^2}{d^2}$$

$$\boxed{\text{Energy Density} = \frac{1}{2} \varepsilon_0 E^2}$$, where $E = \frac{V}{d}$, the Electric field.

DIELECTRICS AND ITS POLARISATION :-

Substance which donot conduct electricity but conduct electrical effects (Electric field) are called dielectrics.

Egs :- Water, H_2, paper, glass, CO_2, HCl etc

Non polar dielectrics :- If the centre of mass of +ve nuclei of a substance coincides with that of the -ve charge ~~thencat~~ the substance is called non polar dielectrics.

Eg :- H_2, CO_2 etc.

Polar Dielectrics :- If the centre of mass of +ve nuclei doesnot coincide with that of the -ve charge, then the substance is called a polar dielectric.

Eg :- HCl, H_2O etc.

Electric polarisation :- The process of converting a non polar molecule into a polar molecule by the application of an electric field is called electric polarisation

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Polarisation Vector :- The dipole moment per unit volume of a substance is called the polarisation vector.

$$P = \frac{p}{V}$$

Electric Susceptibility :- polarisation is directly proportional to the intensity of electric field

ie $P \propto E$

$\therefore$ $\boxed{P = X_e E}$ where X_e is a constant called the Electric Susceptibility.

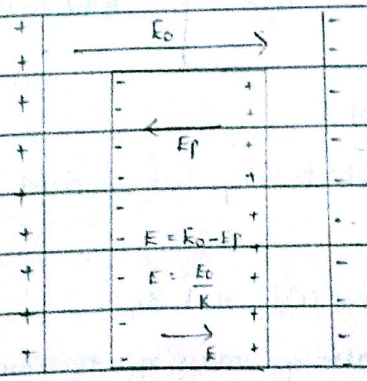
Let $E = 1 N/C$

Then, $P = X_e$

$\therefore$ Electric susceptibility of a material can be defined as the polarisation per unit electric field.

Dielectric strength :- The max value of electric field that can be applied to a dielectric without causing any permanent deformation is called its dielectric strength.

Electric Field inside a diElectric



Let E_0 be the applied electric field & E_p be the field due to polarisation. Then Electric field inside dielectric, E

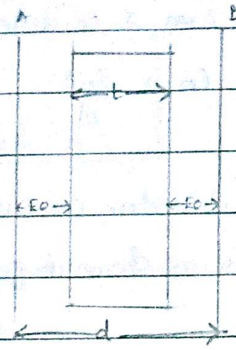
$$E = E_0 - E_p$$

$E = \frac{E_0}{K}$, where K is the dielectric constant of the material.

Effect of introducing a conducting slab between the plates of a capacitor

Consider a ||^{l} plate capacitor of plate area A & distance of separation d. Its' capacitance is given by $C_0 = \frac{\epsilon_0 A}{d}$

Consider a conducting slab of thickness, t introduced in b/w the plates of the capacitor as shown in the fig.



Now the electric field exist only for a distance d-t

$\therefore$ the pot diff b/w the plates of the capacitor,

$$V = E_0 (d-t)$$

$$V = \frac{\sigma}{\varepsilon_0} (d-t)$$

$$V = \frac{Q}{A\varepsilon_0} (d-t)$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{(d-t)}$$

$$C = \frac{\varepsilon_0 A}{d(1-t/d)}$$

$$\boxed{C = \frac{C_0}{(1-t/d)}}$$

ie, the capacitance increases.

Note :- Let the space in b/w the plates of the capacitor is fully filled with a conducting slab.

ie $t=d$

$$C = \frac{C_0}{1-d/d}$$

$$C = \frac{C_0}{0}$$

$$C = \infty$$

ie the capacitance becomes infinite.

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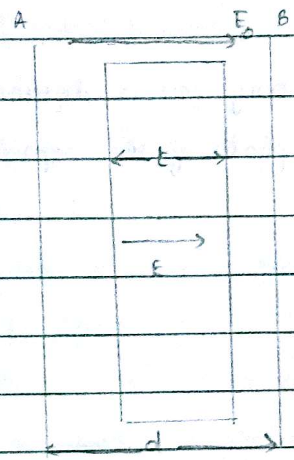
Effect of introducing a dielectric slab in between the plates of a capacitor :-

Consider a || plate capacitor of plate Area A & distance of separation d. Then its capacitance, $C_0 = \frac{\varepsilon_0 A}{d}$.

A dielectric slab of thickness t & dielectric constant K, is inserted in b/w the plates of capacitor as shown in fig.



Let E_0 be the electric field in freespace & E be that in the dielectric. Then

$$E = \frac{E_0}{K}$$

The p.diff. b/w the plates of the capacitor, V

$$V = E_0(d-t) + Et$$

$$= E_0(d-t) + \frac{E_0}{K}t$$

$$= E_0\left((d-t) + \frac{t}{K}\right)$$

$$V = \frac{\sigma}{\varepsilon_0}\left(d-t+\frac{t}{K}\right)$$

$$V = \frac{Q}{A\varepsilon_0}\left(d-t+\frac{t}{K}\right)$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{\left(d-t+\frac{t}{K}\right)}$$

$$C = \frac{A\varepsilon_0}{d-t\left(1-\frac{1}{K}\right)}$$

$$C = \frac{\varepsilon_0 A}{d\left[1 - \frac{t}{d}\left(1 - \frac{1}{k}\right)\right]}$$

$$\boxed{C = \frac{C_0}{\left(1 - t/d\left(1 - 1/k\right)\right)}}$$

Note : Let the space in b/w the plates of the capacitor be completely filled with the dielectric slab. Then,

$t = d$

$$C = \frac{C_0}{\left(1 - d/d\left(1 - 1/k\right)\right)}$$

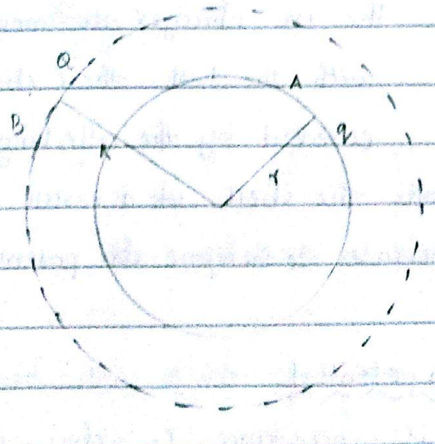
$$= C = \frac{C_0}{1 - 1 + \frac{1}{k}}$$

$$= C = C_0 k$$

$\therefore$ The capacitance becomes k times

Van De Graff Generator : It is a device used to generate high potential.

Principle :- Consider 2 concentric spheres A & B with radii r & R resp. $(r < R)$. Let q be the charge on A and Q be the charge on B. $(Q > q)$.



The electric potential on the surface of the inner shell, V_A

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \longrightarrow ①$$

The potential on the surface of shell B, V_B

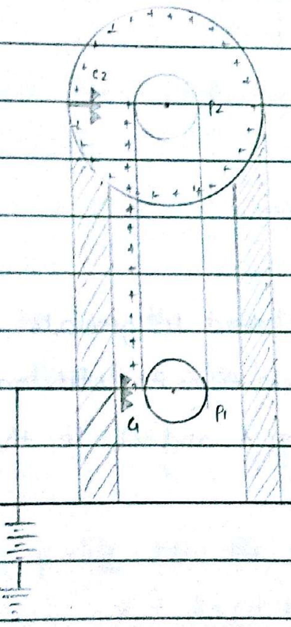
$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \longrightarrow ②$$

From (1) & (2), it is clear that V_A is greater than V_B. ie the inner shell is always at a higher potential than the outer shell, irrespective of the charges given to them. This is the working principle of Van De Graff generator.

Potential diff b/w A & B,

$$V_A - V_B = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Working :-

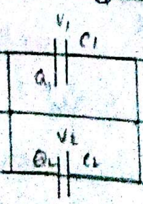


It consists of large metallic sphere fixed on 2 insulated pillars. P_1 & P_2 are 2 pulleys joint by an insulated belt. C_1 & C_2 are 2 metallic combs. C_1 is connected to a voltage source & C_2 is connected to the metal shell.

The comb, C_1 called the spraying comb sprays some +ve charge to the belt. When the belt moves up, the +ve charges also moves along with the belt. These charges will be collected by the collecting comb, C_2 when it reaches inside the shell. As a result, the charge on the metal shell increases & therefore its potential also increases.

→ Expression for common potential :-

Consider a capacitor of capacitance C_1, charge Q_1 & potential V_1, Connected in ||^l to another capacitor of capacitance C_2, Charge Q_2 & potential V_2.



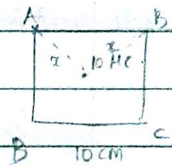
The effective capacitance is given by $C = C_1 + C_2$.
The total charge across the combination of the capacitor, $Q = Q_1 + Q_2$

$CV = C_1V_1 + C_2V_2$, where V is the common potential.

$$V = \frac{C_1V_1 + C_2V_2}{C}$$

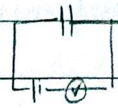
$$\therefore \boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

Q. What is the work done in moving a 2 μC point charge from corner A to corner B of a square ABCD of side 10 cm when a 10 μC charge exist at the centre of the square.



The work done will be zero as the p.diff is zero.
(same distance of separation)

Q The plates of a charged capacitor are connected by a volt meter. If the plates of the capacitors are moved further apart. what will be the effect on the reading of the volt meter.

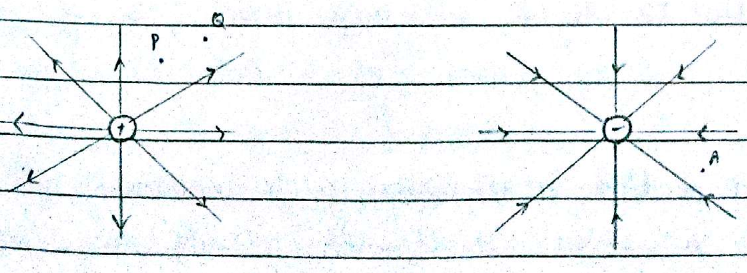


$$\downarrow C = \frac{A\varepsilon_0}{d \uparrow}$$

$$\downarrow C = \frac{Q}{V \uparrow}$$

The reading on the voltmeter increases

Q Fig A & B shows the field lines of a single +ve & -ve charge resp.
imp



i) Give the signs of the potential diff: $V_P - V_Q$ & $V_B - V_A$
ii) Give the sign of the work done by the field in moving a small +ve

charge from q to P

iii) Give the sign of the work done by ~~the field or~~ an external agent in moving a small -ve charge from B to A

i) $V_p - V_q = +ve$

$V_B - V_A = +ve$

ii) The work has to be done against the field

$W = -ve$.

iii) External agent - work done by the agent is in the direction of the force.

$W = +ve$.

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